

A Simple Proof of Grounding Internality

Anonymized

Abstract

Some people think that grounding is a type of identity. And some people think that grounding connections hold necessarily. I show that, under plausible assumptions, if grounding is a type of identity, then grounding connections hold necessarily.

1 Introduction

Some people think of grounding as a type of identity.¹ The idea is that grounds are disjuncts of what they ground. So, A grounds B if and only if B is a disjunction with A as a disjunct. Call this the identity conception of ground. Some people also think grounding connections hold necessarily.² If A grounds B, then necessarily A grounds B. Call this grounding internality. In this paper I show that, under plausible assumptions, the identity conception entails grounding internality. If grounding is a type of identity, then grounding connections hold necessarily.

Why does this matter? Well, *I* think it matters because *I* think grounding is a type of identity. So, this settles whether grounding connections hold necessarily. But, that aside, I think this is good news for grounding internality. It shows it's entailed by an independently motivated conception of ground. And I think it's great news for the identity conception of ground. It shows it generates an independently plausible principle. So this forms a tiny part of an abductive argument for both principles. Together they form a coherent picture, the parts of which are mutually supporting. So I think most people should be a bit more confident in both views by the end of this paper. Anyway,

¹ See [Correia and Skiles \(2017\)](#) and [Lovett \(forthcoming\)](#).

² ([Audi, 2012, 697](#)), ([Fine, 2012, 74](#)) and ([Bennett, 2011, 32](#)) all seem to endorse this. ([Rosen, 2010, 118](#)), [deRosset \(2013\)](#) and ([Dasgupta, 2014, 4](#)) endorse a connected view –grounding necessitarianism– which I'll discuss in §3. [Schaffer \(2010\)](#), [Leuenberger \(2014\)](#), [Skiles \(2015\)](#) and [Litland \(2015\)](#) all oppose these views.

here's the plan. In the next section I'll clarify some conceptual apparatus. In §3 and §4 I'll say more about both internality and the identity conception of ground. In §5 to §7 I'll prove various versions of internality follow from the identity conception of ground.

2 Conceptual Preliminaries

We better start by with some conceptual preliminaries. Let's first fix the relevant notion of identity. We'll call this identification.³ Identification is a bit like ordinary identity. But it connects things in sentence position. Consider the claim 'for water to be wet just is for H₂O to be wet.' This expresses an identification. So does 'For there to be bachelors is for there to be unmarried men' and 'for Cicero to be eloquent is for Tully to be eloquent.' Identification has some general features. It's reflexive, symmetric, and transitive. And it obeys a version of Leibniz's law. If A just is B, then one can replace As for Bs in any (non-opaque) formula without changing the truth value of that formula. So it's a sentential analogue of ordinary identity

Let's now fix the relevant notion of ground. Ground is a kind of non-causal explanation. Consider the relation between sets and their members. The existence of the latter explain that of the former. Or consider the relation between disjunctions and their disjuncts. The truth of the latter explain that of the former. These explanatory connections are grounding connections. In particular, the relevant notion of ground is worldly ground. This contrasts with representational ground. They differ in what conception of identification each fits. Representational ground fits a fine-grained conception. Worldly ground fits a coarse-grained conception.

We can make this contrast precise in a couple ways. On one way, these different conceptions are different *kinds* of identifications.⁴ Representational identification connects representations of the world. Worldly identification connects the states which these represent. So A and A∧A might be worldly identical but representationally distinct. They're different representations of the very same state of affairs. Representational *ground* is then defined as the notion of ground which obeys Leibniz's law for only the

³ See [Dorr \(2016\)](#).

⁴ This seems to me to articulate the approach in [\(Correia, 2017a, 515\)](#) and [\(Correia, 2017b, 58\)](#).

representational notion of identification. Worldly ground obeys it for worldly identification. So the distinction between the two notions of ground is cashed out by this connection to different kinds of identification.

On the second way, these different conceptions of identification are different *theories* of identification. Fine-grained theories reject claims that coarse-grained theories accept. The claim that A just is $A \vee A$ is a good example: the former might accept this whilst the latter will reject it. And depending on your theory of identification, you should endorse different ground-theoretic principles. For example, on the coarse-grained theories, you should reject the claim that A always grounds $A \vee B$. So we can define worldly ground as the notion of ground which conforms to the principles appropriate for the coarse-grained theory. Representational ground conforms to the principles appropriate to the fine-grained theory. I don't have a strong view on how we should draw this distinction. All that matters for this paper is that I'll be talking about worldly ground: a notion of ground fit for identification coarsely conceived.

Finally, we can distinguish between some different kinds of ground.⁵ First, we distinguish between factive and non-factive ground. Factive ground connects only truths. Non-factive ground also connects falsehoods. We define factive ground in terms of non-factive ground: A factively grounds B iff A non-factively grounds B and A obtains. Second, we distinguish between full and partial ground. Full grounds wholly explain what they ground. Partial grounds help explain what they ground. We define partial in terms of full ground: A partially grounds B iff A, perhaps together with some other proposition, fully ground B. Third, we distinguish between strict and weak ground. Nothing strictly grounds itself but *everything* weakly grounds itself. Strict ground is definable in terms of weak ground. A strictly grounds B iff A to weakly ground B and B does not even help weakly ground A. Weak ground is definable in terms of strict ground and identification. A weakly ground B iff A strictly grounds B or A just is B. This now lets us be more precise about the two claims we'll be connecting.

⁵ These are from (Fine, 2012, 51-2).

3 Grounding Internality

Grounding internality is a claim about both strict full ground and weak full ground. The idea is that both connections of full ground hold necessarily. We can distinguish different versions of this claim for non-factive and factive ground. For non-factive ground, the gloss in the introduction holds. Internality says that, if A non-factively grounds B, then necessarily A non-factively grounds B. For factive ground, internality says something mildly more complicated. It says that if A factively grounds B, then necessarily, if A is true, then A factively grounds B. These two claims are connected by entailment: the non-factive claim entails the factive claim.⁶ But they're also connected by flavour: both make a necessity claim about connections of ground. I'm going to argue that the identity conception of ground entails both these claims.

Internality should be distinguished from a closely connected claim: grounding necessitarianism. Necessitarianism says that, if A fully factively or non-factively grounds B, then necessarily, if A is true, then B is true. The proofs in §5–§7 will be proofs of grounding internality. But these claims aren't independent. Under a very plausible assumption, grounding internality entails grounding necessitarianism. The assumption is that full ground implies material implication: if A fully grounds B, then if A is true then B is true. As far as I know, nobody has rejected this claim in print. I think that's for good reason: any notion which violates this claim just doesn't seem to be the notion which was introduced by seminal works on ground.⁷ But with this in hand, necessitarianism follows from internality in any plausible modal logic.⁸ So, if the identity conception implies internality, it implies necessitarianism. So the proofs to come focus on internality.

What's been previously said about these claims? Counter-examples have been offered to both internality and necessitarianism. The case of universal generalization is representative.⁹ Several people have suggested that any universal generalization $\forall xFx$,

⁶ I show this in §7.

⁷ See especially Fine (2001, 2012) and Rosen (2010).

⁸ Here a proof for strict full non-factive ground ('<'). Suppose $(A < B) \rightarrow (A \rightarrow B)$. This is the claim that ground is at least as strong as material implication. By the rule of necessitation, it follows from this that $\Box((A < B) \rightarrow (A \rightarrow B))$. From this it follows by **K** that $\Box(A < B) \rightarrow \Box(A \rightarrow B)$. But internality says that $A < B \rightarrow \Box(A < B)$. So, by classical logic, $A < B \rightarrow \Box(A \rightarrow B)$. And this is just necessitarianism.

⁹ See Bader (manuscript), Chudnoff (manuscript) and (Skiles, 2015, 730) for discussion of this case.

is grounded in the plurality of its instances, Fa, Fb, \dots . But these conjoined instances don't strictly imply the universal generalization. This is meant to refute a plural version of both claims. The plural version of grounding internality says that, if some plurality of sentences Δ grounds B , then necessarily their conjunction grounds B . The plural version of necessitarianism says that, if some plurality of sentences Δ grounds B , then their conjunction strictly implies B . If universal generalizations are fully grounded in their instances, then they provide a counterexample to these claims.

But there's a standard solution this counter-example. The solution is to say that universal generalizations are grounded in their instances together with a totality fact. The totality fact is just a fact that says, of some list of objects, that they're all the things there are.¹⁰ Of course, some people have qualms about totality facts.¹¹ But the possibility of this solution makes the counter-example indecisive. This dialectic is common with counterexamples to internality and necessitarianism. Defenders of these claims can insist that putative counterexamples just identify a partial ground. The full ground included things which do strictly imply the grounded fact. That doesn't mean such counter-examples have no weight: the initial intuitions about ground they rely on retain some plausibility. But it means that they're not decisive.¹²

But what has been said in defense of these claims? Not all that much. Many people have endorsed internality.¹³ And many people have endorsed necessitarianism.¹⁴ But few have provided extended argument for either. The only paper-length argument of which I know is in Trogdon (2013).¹⁵ He thinks ground and essence are intimately connected. In particular, he thinks that if A grounds B , then it must be essential to

¹⁰ This is the view in (Fine, 2012, 61).

¹¹ (Skiles, 2015, 736) uses such qualms to reject this solution.

¹² Litland (2015) has a counterexample to internality which can't be given this treatment. But his counterexample relies on a few important assumptions. First, he assumes that disjunctions are always strictly grounded in at least one of their disjuncts. Second, he assumes that existential generalizations are always strictly grounded in at least one of their instances. Third, he assumes that A always strictly partially grounds the fact that ' A ' is true. Yet, on the worldly conception, assumptions like this must be restricted. Consider just the first. On the worldly conception, we accept that A just is $A \vee A$. So we cannot accept this in full generality. $A \vee A$ is not, on the worldly conception, grounded in either of its disjuncts. Yet there is not yet consensus on how exactly to restrict these rules. So it remains to be seen whether Litland's counterexample works on the rules ultimately appropriate for worldly ground. It's currently only compelling for representational ground.

¹³ See (Audi, 2012, 697), (Fine, 2012, 74) and (Bennett, 2011, 32).

¹⁴ For example, see deRosset (2010), (Rosen, 2010, 118) and (Dasgupta, 2014, 4).

¹⁵ There are some other relevant, shorter, arguments in the literature. See (Skiles, 2015, 736-47) for a general survey.

A, B or the plurality A, B that A materially implies B. But such essential facts holds necessarily. So grounding necessitarianism follows. Trogdon has an argument for the existence of this intimate connection. He thinks it explains the putative fact that, were we fully informed about the essences of the ground and grounded facts, we would know that if A obtains then B obtains. I'm not going to critique this argument here. I just want to point out we probably shouldn't take Trogdon to have settled the matter. That's because the specific connection between ground and essence Trogdon relies on is highly controversial. An argument which relies on such a connection shouldn't make us sure of grounding necessitarianism.

So the truth of both internality and necessitarianism remains open. I'll argue that adopting the identity conception closes it. But let's now say more about this conception.

4 The Identity Conception of Ground

According to the identity conception of ground, grounding is a kind of identity. In particular, grounded facts are disjuncts of the facts they ground. [Correia and Skiles \(2017\)](#) argue for this view. They argue that A grounds B just in case there's some C such that A just is $A \vee C$. ([Correia, 2010, 270-73](#)) and [Lovett \(forthcoming\)](#) both advance an instance of this claim. They both suggest that A grounds B just in case B just is $A \vee B$. We'll work with the second claim. But that's just to make the proofs in §5–§7 simpler: there's little practical difference between the two. They're both entailed by principles we'll discuss in a moment. And, depending on the background logic of ground, they turn out to be equivalent.¹⁶ So, for our purposes, we can count these as the same conception of ground.

We'll want a formal statement of the view. Let ' \leq ' stand for weak full non-factive ground. Let ' \equiv ' stand for identification. We'll treat these both as sentential connectives. Then the formal statement is:

$$\mathbf{I} \quad A \leq B \leftrightarrow (B \equiv (B \vee A))$$

In the rest of this section I want to explain why anyone would believe this. I'm only

¹⁶ They are in the logic presented in [Lovett \(forthcoming\)](#).

going to give one reason. But it's a good one. This view follows from some plausible principles. The principles are:

(i)	$((A \equiv B) \wedge (B \equiv C)) \rightarrow (A \equiv C)$	Transitivity of \equiv
(ii)	$A \vee A \equiv A$	Idempotence of \vee
(iii)	$B \leq B \vee A$	\vee -I ₁
(iv)	$A \leq B \vee A$	\vee -I ₂
(v)	$A \leq B \rightarrow ((C \vee A) \leq (C \vee B))$	\vee -supplementation for \leq
(vi)	$((A \leq B) \wedge (B \equiv C)) \rightarrow A \leq C$	Substitution ₁
(vii)	$((A \leq C) \wedge (B \equiv C)) \rightarrow A \leq B$	Substitution ₂
(viii)	$((A \leq B) \wedge (B \leq A)) \rightarrow A \equiv B$	Antisymmetry

Here's a proof:¹⁷

Left-to-right. Suppose $A \leq B$. Then by (v) it follows that $B \vee A \leq B \vee B$.

By (ii), we have $B \vee B \equiv B$. So by (vi), it follows that $B \vee A \leq B$. Yet by

(iii) we have $B \leq B \vee A$. So by (viii) it follows that $B \equiv B \vee A$.

Right-to-left. Suppose $B \equiv B \vee A$. By (iv), $A \leq B \vee A$. So by (vii), $A \leq B$.

As I said, I think these principles are plausible. (i), (vi) and (vii) seem essential to identification being an *identity*-like connection. Anything like identity conforms to principles like this. (iii) and (iv) capture the claim that disjunctions are (weakly) grounded in their disjuncts. This is endorsed by every extant logic of ground which deals with weak ground. (ii) and (v) might be disputed.¹⁸ But remember we're working with a worldly notions of both ground and identification. For this notion these seem plausible. It seems plausible, for instance, that A and $A \vee A$ are just different representations of the very same state of affairs.

That leaves (viii). Yet there's a simple argument for (viii). In §2 I said A weakly fully grounds B iff A strictly grounds B or A just is B . So suppose A weakly fully grounds B and B weakly fully grounds A . Then either A strictly fully grounds B or A just is B . But A can't strictly ground B . That's because B weakly fully grounds A . So A just is B . So (viii) follows straightforwardly. So all these principles are defensible. Now I don't think that makes the above argument decisive. One could resist some of these

¹⁷ This kind of proof comes from (Correia and Skiles, 2017, 19). But, for novelties sake, the two are a little different: they rely on the symmetry rather than the transitivity of identification.

¹⁸ For dispute, see Dorr (2016).

principles anyway. But it means the identity conception isn't crazy: a case can be made for it. So we can now move on to the proofs. Let's see how the identity conception entails grounding internality.

5 Weak Full Non-Factive Ground

We'll start with weak full non-factive ground. For this notion, we formulate grounding internality as follows:

$$\mathbf{GI}(\leq) \quad A \leq B \rightarrow \Box(A \leq B)$$

In this section, we'll derive this from **I**. To do we'll need some further assumptions. The most important is that identifications hold necessarily. In other words:

$$\text{Necessity of Identification} \quad A \equiv B \rightarrow \Box(A \equiv B)$$

What's to be said for this? I think the most compelling argument invokes Leibniz's Law.¹⁹ Here it is. Suppose *A* just is *B*. It's necessary that *A* just is *A*. Everything has to be the same as itself. Now suppose that *A* just is *B*. Then, by Leibniz's Law (for identification), it must be necessary that *A* just is *B*. So if *A* just is *B*, then necessarily *A* just is *B*.

Some people do reject arguments like this.²⁰ So, if you're such a person, you should interpret this paper as showing the connection between three claims rather than two. But I myself think this argument settles the matter. So I'll assume the necessity of identifications throughout. We now need just two more assumptions. They are:

$$\begin{array}{ll} \mathbf{K} & \Box(A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B) \\ \mathbf{NEC} & A/\Box A \end{array}$$

NEC is, of course, restricted to theorems. Every plausible logic of metaphysical modality contains these principles. So they should be uncontroversial. So here's the simple

¹⁹ Barcan (1947) and Kripke (1971) are important sources of this sort of argument.

²⁰ See e.g. Gibbard (1975) and Gray (2001).

proof:

(1)	$A \leq B$	Assumption
(2)	$A \leq B \rightarrow B \equiv B \vee A$	GI , CL
(3)	$B \equiv B \vee A$	MP; 1, 2
(4)	$B \equiv B \vee A \rightarrow \Box(B \equiv B \vee A)$	Necessity of Identification
(5)	$\Box(B \equiv B \vee A)$	MP; 3, 4
(6)	$(B \equiv B \vee A) \rightarrow (A \leq B)$	GI , CL
(7)	$\Box(B \equiv B \vee A) \rightarrow \Box(A \leq B)$	NEC, K, CL; 6
(8)	$\Box(A \leq B)$	MP; 5, 7
(9)	$A \leq B \rightarrow \Box(A \leq B)$	Conditional Proof; 1, 8

‘MP’ indicates where I use modus ponens. ‘CL’ indicates where I use unspecified rules from classical propositional logic. The numbers to the left of the semi-colon say what prior lines in the proof we use to derive the present line. So, this proof shows that, given our assumptions, **I** entails **GI**(\leq). If grounding is a type of identity, then connections of weak full non-factive ground hold necessarily.

6 Strict Full Non-Factive Ground

Let’s now turn to strict full non-factive grounds. We’ll use ‘<’ to stand for this notion.

The version of internality for this notion is:

$$\mathbf{GI}(<) \quad A < B \rightarrow \Box(A < B)$$

In this section I’ll prove that, if grounding is a type of identity, then connections of strict full non-factive ground hold necessarily. I’ll give two different proofs. The first relies on stronger modal assumptions. The second relies on stronger ground-theoretic assumptions. Both rely on **GI**(\leq). Both also rely on the necessity of distinctness. So this seems core to establishing internality for strict ground.

6.1 First Proof

We begin with some definitions. Let \leq stand for weak partial ground. Let $(A \not\leq B)$ stand for $\neg(A \leq B)$. Let Δ be a finite plurality of sentences. Let $\hat{\Delta}$ be the result of conjoining

all those sentences, starting from the left. Let $A^{[B/\Phi]}$ be the result of replacing some occurrences of B in A for Φ . Then the principles on which the first proof relies are:

GROUND-THEORETIC ASSUMPTIONS

Definition of ($<$)	$A < B \leftrightarrow ((A \leq B) \wedge (B \not\leq A))$
Definition of (\leq) I	$A \leq B \leftrightarrow \exists p(A, p \leq B)$
I*	$\Delta \leq B \leftrightarrow B \equiv B \vee \hat{\Delta}$

HIGHER-ORDER QUANTIFICATION ASSUMPTIONS

$\forall\exists$ – Duality	$\forall p\neg A \leftrightarrow \neg\exists pA$
\forall -Elimination	$\forall pA/A^{[B/\Phi]}$
\forall -Introduction	$A/\forall pA^{[B/p]}$

Where, in \forall -I, B cannot occur in any undischarged assumptions.

MODAL ASSUMPTIONS

Barcan Formula	$\forall p\Box A \rightarrow \Box\forall pA$
Necessity of Distinctness	$A \neq B \rightarrow \Box(A \neq B)$

And the proof is:

(1)	$A < B$	Assumption
(2)	$A \leq B$	Definition of ($<$), CL; 1
(3)	$\Box(A \leq B)$	GI (\leq), CL; 2
(4)	$B \not\leq A$	Definition of ($<$), CL; 1
(5)	$B \leq A \leftrightarrow \exists p(B, p \leq A)$	Definition of (\leq) I
(6)	$B \not\leq A \leftrightarrow \neg\exists p(B, p \leq A)$	CL; 5
(7)	$\neg\exists p(B, p \leq A)$	MP; 4, 6
(8)	$\forall p(B, p \not\leq A)$	$\forall\exists$ -Duality, CL; 7
(9)	$B, C \not\leq A$	\forall -Elimination; 8
(10)	$A \neq (A \vee ((B \wedge C)))$	I* , CL; 9
(11)	$\Box(A \neq (A \vee ((B \wedge C)))$	Necessity of Distinctness, CL; 10
(12)	$\Box(B, C \not\leq A)$	I* , NEC, K, CL; 11
(13)	$\forall p\Box(B, p \not\leq A)$	\forall -Introduction; 12
(14)	$\Box\forall p(B, p \not\leq A)$	Barcan Formula, CL; 13
(15)	$\Box\neg\exists p(B, p \leq A)$	$\forall\exists$ -Duality, CL, NEC, K; 14
(16)	$\Box(B \not\leq A)$	Definition of (\leq) I, NEC, K, CL; 15
(17)	$\Box(A \leq B \wedge B \not\leq A)$	CL, NEC K; 4, 16
(18)	$\Box(A < B)$	Definition of ($<$), NEC, K, CL; 17
(19)	$A < B \rightarrow \Box(A < B)$	CP; 1, 18

Let's discuss the assumptions. The first two ground-theoretic assumptions are our definitions of strict full ground and weak partial ground. These are anodyne. The third,

\mathbf{I}^* , is a strengthening of \mathbf{I} . It extends it to cases in which several propositions together ground something. If you think that singular ground is a type of identity, it's hard to believe that plural ground is not. It's hard to credit that facts of the form $(A \leq B)$ are equivalent to identifications, but those of the form $(\Delta \leq B)$ aren't. So \mathbf{I}^* , or some comparable principle, seems plausible.

So let's look at the other assumptions. The rules for higher-order quantification are entirely standard. If you really don't like sentential quantification, maybe you'll reject these. So we'll see how to do without them in the second proof. The Barcan Formula is controversial. But note that this is a higher-order version of the Barcan Formula. Even if one objects to the first-order Barcan Formula there's good reason to accept its higher-order counterparts.²¹ But it's controversial nonetheless. So we'll see how to do without it in our second proof. Finally, we have the necessity of distinctness. This says that if for A to be the case isn't just for B to be the case, then necessarily for A to be the case isn't just for B to be the case. This follows from the necessity of identity and the B-Schema.²² So it's rarely denied. As such, I think the assumptions in this argument are pretty plausible. But it's worth seeing a proof which doesn't rely on the more controversial ones.

6.2 Second Proof

Let's turn to that proof. This proof relies on the following principle:

$$\text{Definition of } (\leq) \text{ II} \quad A \leq B \leftrightarrow A, B \leq B$$

This says that if A partially grounds B, then A together with B itself fully ground B. I'll explain why this is plausible in a moment. But first let's see how this gives us a proof of $\mathbf{GI}(\leftarrow)$. Here's the proof:

²¹ Or so (Williamson, 2013, 262-300) persuasively argues.

²² The B-Schema says that $A \rightarrow \Box \Diamond A$.

(1)	$A < B$	Assumption
(2)	$A \leq B$	Definition of ($<$), CL; 1
(3)	$\Box(A \leq B)$	GI (\leq), CL; 2
(4)	$B \not\leq A$	Definition of ($<$), CL; 1
(5)	$B \leq A \leftrightarrow B, A \leq A$	Definition of (\leq) II
(6)	$B, A \not\leq A$	CL; 4, 5
(7)	$A \neq (A \vee (B \wedge A))$	I* , CL; 6
(8)	$\Box(A \neq (A \vee ((B \wedge A))))$	Necessity of Distinctness, CL; 7
(9)	$\Box(B, A \not\leq A)$	I* , NEC, K, MP; 8
(10)	$\Box(B \not\leq A)$	Definition of (\leq) II, NEC, K MP; 9

One then completes the proof with line 17-19 of the previous proof. So we can dispense with the Barcan formula and with the rules for sentential quantifiers. Note that a proof like this goes through if if *anything* of the form $A \leq B \leftrightarrow \Phi$ is true, where Φ contains just ' \leq ' flanked by truth-functional formulae. We don't need the particular definition of weak partial ground above. We just need there to be some definition of weak partial ground in terms of weak full ground and truth functors.

But why think this definition in particular is plausible? Because it follows from the following three principles:

Identity	$B \leq B$
Subsumption(\leq/\leq)	$A, B \leq B \rightarrow A \leq B$
Amalgamation (\leq/\leq)	$(A \leq C \wedge B \leq C) \rightarrow A, B \leq C$

Identity is definitional of weak fully ground. Subsumption(\leq/\leq) is definitional of weak partial ground. So Amalgamation (\leq/\leq) is the only contestable one of these principles. Amalgamation (\leq/\leq) says that we can add a partial ground of C to any full ground of C and get another full ground of C. This isn't obviously true. But I think it's plausible. Suppose B fully grounds C and A helps ground C. Then why would adding A to B destroy the connection of full ground? Why should adding something explanatorily relevant to a full explanation of C eliminate that explanation? I doubt it would.²³ If the reader agrees, then they should endorse this amalgamation principle. From this follows the second definition of weak partial ground. And this means **I*** entails **GI**($<$).

²³ It's worth noting that, on certain extremely fine-grained notions of ground, no amalgamation principle is plausible. Correia (2014) considers such a notion: logical grounding. In the system he there presents, $\neg\neg\neg p$ is fully grounded by p and fully grounded by $\neg\neg p$. But it's not grounded by the plurality $p, \neg\neg p$. So no amalgamation rule is sound in his system. Why is that? At root, it's because there's no cut rule in the system he presents there. But a cut rule is extremely plausible for other notions of ground. So this shouldn't make us sceptical of amalgamation rules in the case of worldly ground.

7 Strict Full Factive Ground

So far we've just talked about non-factive ground. Let's end by extending these proofs to factive ground. In particular, let's extend them to strict full factive ground.

There's a couple of ways we could define this notion. The first closely follows the §2 gloss on factive ground. We say A strictly fully factively ground B when A obtains and A strictly fully *non*-factively grounds B. Let's use ($<_f$) to express the first notion. Formally, this corresponds to:

$$\text{Definition of } <_f \quad A <_f B \leftrightarrow (A \wedge (A < B))$$

The second closely follows the §2 gloss on strict ground. We first define a notions of *weak* full factive ground. We say A weakly fully factively grounds B when A obtains and A weakly fully *non*-factively grounds B. We then define a notion of weak *partial* factive ground. We say A weakly partially factively grounds B when there's some p such that A, p weakly fully factively ground B. We then say A strictly fully factively grounds B iff A weakly fully factively grounds B and B does not weakly partially factively ground B. Let's use ($<_f^*$) to express this notion. Formally, this corresponds to:

$$\text{Definition of } <_f^* \quad A <_f^* B \leftrightarrow (A \wedge ((A \leq B) \wedge \neg \exists p(p \wedge (p, A \leq B))))$$

So we have two definitions of strict full factive ground. Which of these should we choose? Fortunately, we need make no choice. Given the assumptions we've already made, they're equivalent.²⁴ This is, I think, another nice consequence of our assump-

²⁴ Proving this means proving that $A <_f B$ iff $A <_f^* B$. Here's the proof:

Left-Right: Suppose $A <_f B$. From this, we use the definition of ($<_f$), to derive $(A \wedge (A < B))$. We then use the definition of ($<$) to derive $(B \leq A)$. We then, for reductio, suppose that $\exists p(p \wedge (p, B \leq A))$. From this, by existential introduction, we derive $C \wedge (C, B \leq A)$. From this we derive $C, B \leq A$. Then, by the definition of (\leq) II, we derive $B \leq A$. This conflicts with $(B \leq A)$, which we already established. So we can use apply reductio to infer that $\neg \exists p(p \wedge (p, B \leq A))$. Now we use the already-derived $(A \wedge (A < B))$, and the definition of ($<$), to derive $(A \wedge (A \leq B))$. From this it follows by the definition of ($<_f^*$) that $A <_f^* B$.

Right-Left: Suppose $A <_f^* B$. Then we use the definition of ($<_f^*$) to derive A . We now suppose, for reductio, that $B \leq A$. Then, by the definition of (\leq) II, we derive that $A, B \leq A$. Since we already derived A , it follows that $A \wedge (A, B \leq A)$. So by existential generalization, it follows that $\exists p(p \wedge (p, B \leq A))$. Yet from $A <_f^* B$ and the definition of ($<_f^*$) we can derive $\neg \exists p(p \wedge (p, B \leq A))$. So we can apply reductio, and infer that $B \not\leq A$. Now, from our initial supposition and the definition of ($<_f^*$), we derive $(A \wedge (A \leq B))$. Then we derive $(A \wedge (A \leq B)) \wedge (B \not\leq A)$. Then, by the definition of ($<$), we derive $(A \wedge (A < B))$. So, by the definition

tions. And it means we need only consider a version of internality for ' $<_f$ '. That version is:

$$\mathbf{GI}(<_f) \quad A <_f B \rightarrow \Box(A \rightarrow A <_f B)$$

We want to prove this follows from the identity conception of ground. To do this, we'll prove it follows from $\mathbf{GI}(<)$. Here's the proof:

- | | | |
|-----|---|--------------------------|
| (1) | $A <_f B$ | Assumption |
| (2) | $A < B$ | Definition of $<_f$, CL |
| (3) | $\Box(A < B)$ | $\mathbf{GI}(<)$, CL; 2 |
| (4) | $A <_f B \rightarrow \Box(A < B)$ | Conditional Proof; 1, 3 |
| (5) | $(A \wedge (A < B)) \rightarrow A <_f B$ | Definition of $<_f$ |
| (6) | $(A < B) \rightarrow (A \rightarrow (A <_f B))$ | CL; 5 |
| (7) | $\Box(A < B) \rightarrow \Box(A \rightarrow (A <_f B))$ | NEC, K, MP; 6 |
| (8) | $A <_f B \rightarrow \Box(A \rightarrow A <_f B)$ | CL; 4, 7 |

And we're done: this proof is easily adapted to the other factive full notions of ground. So versions of internality holds for these notions too.

8 Conclusion

Let's sum up what we've seen. We've looked at many notions of ground. For each notion, we've seen that the identity conception of ground entails grounding internality. So, if grounding is a kind of identity, then grounding connections hold necessarily. I believe the identity conception. So I think this settles whether grounding internality is true. But, even if you don't agree with that, this is good news for both internality and the identity conception. They're stronger together than they would be apart.

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Note here we rely on the second definition of (\leq) II. This followed from Amalgamation (\leq/\leq) , together with some other rules. So the best way to resist the equivalence claim, it seems to me, is to deny Amalgamation (\leq/\leq) .

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